An Elementary Solution to the IBM August 1998 Puzzle

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March 1, 2005

This short article provides an elementary solution for the IBM August 1998 Puzzle (see `http://domino.research.ibm.com/Comm/wwwr_ponder.nsf/challenges/August1998.html`).

Solution. We will use the following theorem.

Theorem 0.1 *In a triangle* ABC, $|AB| > |BC|$ *iff* $\angle BCA > \angle BAC$, *where* $\angle BCA$ *is the angle between* CB *and* CA.

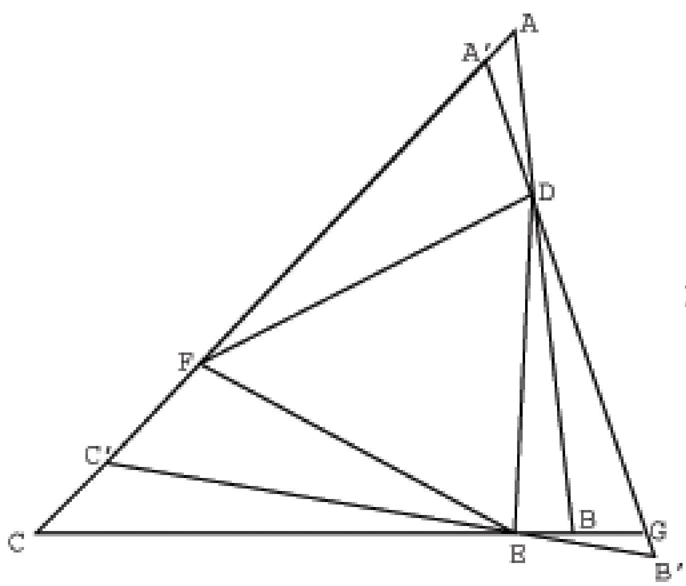

Fig. 1: picture for cc

Proof of the puzzle. We will prove that each of $\angle CAB$, $\angle ABC$ and $\angle BCA$ is no less than 60°. Assume that $\angle CAB < 60^\circ$. Then because $\angle CFD > 60^\circ$, we can find a point A' on line segment AF such that $\angle CA'D = 60^\circ$. Because $\angle AA'D = 120^\circ > 60^\circ > \angle CAB = \angle A'AD$, we have $|AD| > |A'D|$ by the theorem. Then $|CF| = |AD| > |A'D|$, and we can find a point C' on line segment CF such that $|C'F| = |A'D|$. It is easy to show the triangles $A'FD$ and $C'EF$ are congruent (note that both $\angle A'DF$ and $\angle C'FE$ equal $180^\circ - 60^\circ - \angle A'FD$), so $\angle A'C'E = 60^\circ$. Extend line segments $C'E$ through E and $A'D$ through D and denote their intersection B'. Then $A'B'C'$ is an equilateral triangle and $|A'D| = |C'F| = |B'E|$. Extend line segment CB through B until it meets line segment $A'B'$ at a point G. Because $\angle EGB' = \angle B'A'C' + \angle A'CG > \angle B'A'C' = \angle GB'E = 60^\circ$, we have $|EB'| > |EG| > |EB|$ by the theorem. However, we also have $|EB| = |AD| > |A'D| = |EB'|$, which is a contradiction. Therefore, the assumption that $\angle CAB < 60^\circ$ is incorrect. $\square$