# An Efficient and Flexible Approach to Resolution Proof Reduction

**S.F. Rollini**, R. Bruttomesso and N. Sharygina

Formal Verification and Security Group

University of Lugano

October 7th, 2010

# Outline

# Outline

# Outline

# Outline

# Outline

# Background
## SAT

- Propositional SAT $p \wedge (\overline{p} \vee r)$

# Background
## SAT

- Propositional SAT $p \wedge (\overline{p} \vee r)$
- Propositional proof of unsatisfiability

# Background
## SAT

- Propositional SAT $p \wedge (\overline{p} \vee r)$
- Propositional proof of unsatisfiability
  - Certificate of unsatisfiability for boolean formula
  - Generated by logging steps at solving time

# Background
## SAT

- Propositional SAT $p \wedge (\overline{p} \vee r)$
- Propositional proof of unsatisfiability
  - Certificate of unsatisfiability for boolean formula
  - Generated by logging steps at solving time
- DPLL SAT solver [Davis60,62]

# Background
## SAT

- Propositional SAT $p \wedge (\overline{p} \vee r)$
- Propositional proof of unsatisfiability
  - Certificate of unsatisfiability for boolean formula
  - Generated by logging steps at solving time
- DPLL SAT solver [Davis60,62]
  - Search space boolean assignments
  - Backtracking
  - Learning [Marques-Silva96]
  - Proof as combination sequence subproofs

# Background

## Resolution System

- Literal $p \quad \overline{p}$

# Background

## Resolution System

- Literal $p \quad \overline{p}$
- Clause $p \vee \overline{q} \vee r \vee \ldots \quad \rightarrow \quad p\overline{q}r \ldots$
- Empty clause $\bot$

# Background

## Resolution System

- Literal $p \quad \overline{p}$
- Clause $p \vee \overline{q} \vee r \vee \ldots \quad \rightarrow \quad p\overline{q}r\ldots$
- Empty clause $\bot$
- Resolution rule $\dfrac{pC \qquad \overline{p}D}{CD}\, p$

  Antecedent Resolvent Pivot

# Background

## Resolution System

- Literal $p \quad \overline{p}$
- Clause $p \vee \overline{q} \vee r \vee \ldots \quad \rightarrow \quad p\overline{q}r \ldots$
- Empty clause $\bot$
- Resolution rule $\dfrac{pC \qquad \overline{p}D}{CD}\, p$

  Antecedent Resolvent Pivot

- Resolution proof of unsatisfiability of a set of clauses $S$

# Background

## Resolution System

- Literal $p \quad \overline{p}$
- Clause $p \vee \overline{q} \vee r \vee \ldots \quad \rightarrow \quad p\overline{q}r \ldots$
- Empty clause $\bot$
- Resolution rule $\dfrac{pC \qquad \overline{p}D}{CD}\, p$

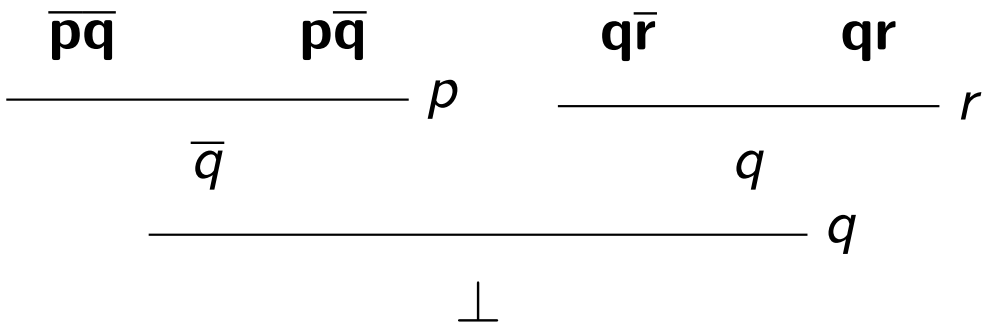
  Antecedent Resolvent Pivot

- Resolution proof of unsatisfiability of a set of clauses $S$
  - Tree
  - Leaves as clauses of $S$
  - Intermediate nodes as resolvents
  - Root as unique empty clause

# Resolution Proofs

## Example

- Set of clauses $\{\overline{pq}, p\overline{q}, q\overline{r}, qr\}$
- Proof of unsatisfiability



# SMT

## Satisfiability Modulo Theories

- More expressivity w.r.t. boolean logic

# SMT

## Satisfiability Modulo Theories

- More expressivity w.r.t. boolean logic
  - Timed automata, hybrid systems, ...
  - Arbitrary precision arithmetic
  - Data structures

# SMT

## Satisfiability Modulo Theories

- More expressivity w.r.t. boolean logic
  - Timed automata, hybrid systems, ...
  - Arbitrary precision arithmetic
  - Data structures
- Satisfiability w.r.t. background theory

# SMT

## Satisfiability Modulo Theories

- More expressivity w.r.t. boolean logic
  - Timed automata, hybrid systems, ...
  - Arbitrary precision arithmetic
  - Data structures
- Satisfiability w.r.t. background theory
- Example $p \wedge ((a + b \leq 0) \vee \overline{p}) \wedge (a = 1)$

# SMT

## Satisfiability Modulo Theories

- More expressivity w.r.t. boolean logic
  - Timed automata, hybrid systems, ...
  - Arbitrary precision arithmetic
  - Data structures
- Satisfiability w.r.t. background theory
- Example $p \wedge ((a + b \leq 0) \vee \overline{p}) \wedge (a = 1)$
- SMT solver

# SMT

## Satisfiability Modulo Theories

- More expressivity w.r.t. boolean logic
  - Timed automata, hybrid systems, ...
  - Arbitrary precision arithmetic
  - Data structures
- Satisfiability w.r.t. background theory
- Example $p \wedge ((a + b \leq 0) \vee \overline{p}) \wedge (a = 1)$
- SMT solver
  - DPLL SAT solver
  - Theory solver

# Outline

# Proof Reduction

## Motivation

- Resolution proofs find application in several ambits

# Proof Reduction

## Motivation

- Resolution proofs find application in several ambits
  - Interpolation-based model checking
  - Abstraction techniques
  - Unsatisfiable core extraction in SAT/SMT
  - Automatic theorem proving

# Proof Reduction

## Motivation

- Resolution proofs find application in several ambits
  - Interpolation-based model checking
  - Abstraction techniques
  - Unsatisfiable core extraction in SAT/SMT
  - Automatic theorem proving
- Problems

# Proof Reduction

## Motivation

- Resolution proofs find application in several ambits
  - Interpolation-based model checking
  - Abstraction techniques
  - Unsatisfiable core extraction in SAT/SMT
  - Automatic theorem proving

- Problems
  - Size affects efficiency
  - Size can be exponential w.r.t. input formula

# Proof Reduction

## Motivation

- Resolution proofs find application in several ambits
  - Interpolation-based model checking
  - Abstraction techniques
  - Unsatisfiable core extraction in SAT/SMT
  - Automatic theorem proving
- Problems
  - Size affects efficiency
  - Size can be exponential w.r.t. input formula
- Reduction/compression of resolution proofs is important

# Related Work

## Features

- Post-processing approach

# Related Work

## Features

- Post-processing approach
- SAT/SMT solving framework

# Related Work

## Features

- Post-processing approach
- SAT/SMT solving framework
  - Explicit DPLL
  - Generic

# Related Work

## Features

- Post-processing approach
- SAT/SMT solving framework
  - Explicit DPLL
  - Generic
- Compression techniques

# Related Work

## Features

- Post-processing approach
- SAT/SMT solving framework
  - Explicit DPLL
  - Generic
- Compression techniques
  - Clauses subsumption checking [Amjad07]
  - Proof reordering based on literals linking [Amjad07]
  - Proof reordering based on variable splitting [Cotton10]
  - Merging of shared substructures in subproofs [Sinz07]
  - Memoization of shared substructures [Amjad08,Cotton10]
  - Algebraic approach, resolution hypergraphs [Fontaine10]
  - Removal pivots redundancies along paths [Bar-Ilan08]

# Pivots Redundancies [Bar-Ilan08]

- No need to resolve more than once on a pivot in a path leaf-root

# Pivots Redundancies [Bar-Ilan08]

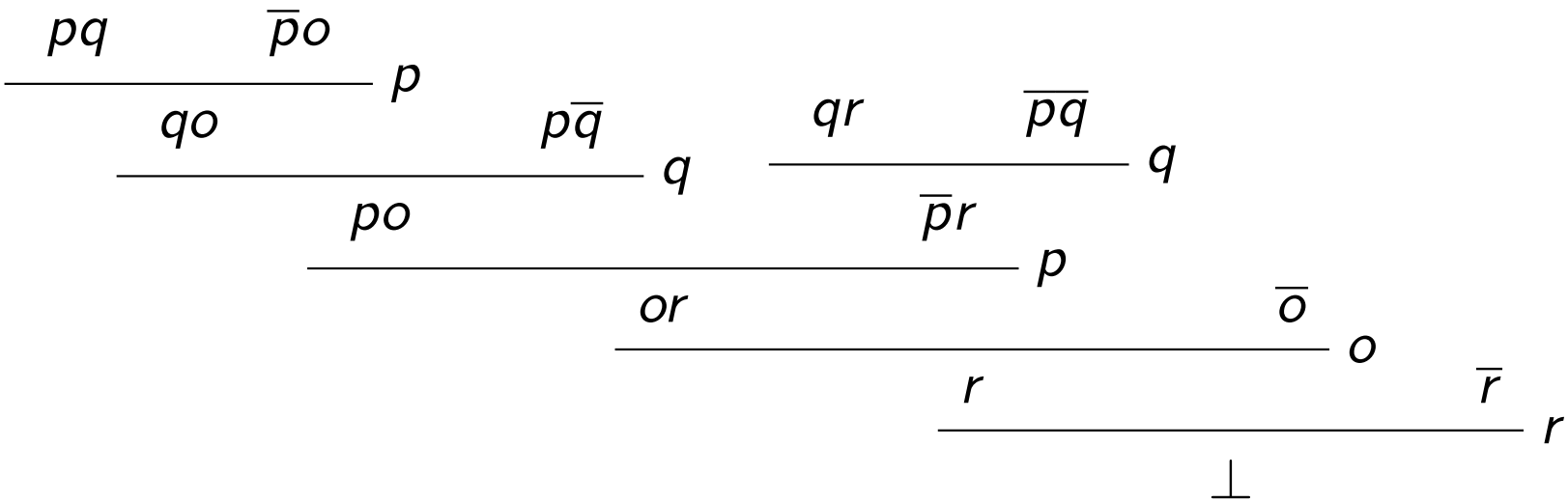
- No need to resolve more than once on a pivot in a path leaf-root
- O.Bar-Ilan, O.Fuhrmann, S.Hoory, O.Shacham and O.Strichman: RecyclePivots

# Pivots Redundancies [Bar-Ilan08]

- No need to resolve more than once on a pivot in a path leaf-root
- O.Bar-Ilan, O.Fuhrmann, S.Hoory, O.Shacham and O.Strichman: RecyclePivots
  - Perform DFS from root to leaves
  - Track pivots occurrences along paths
  - In case of multiple occurrences keep the closest one to root
  - Output regular proof

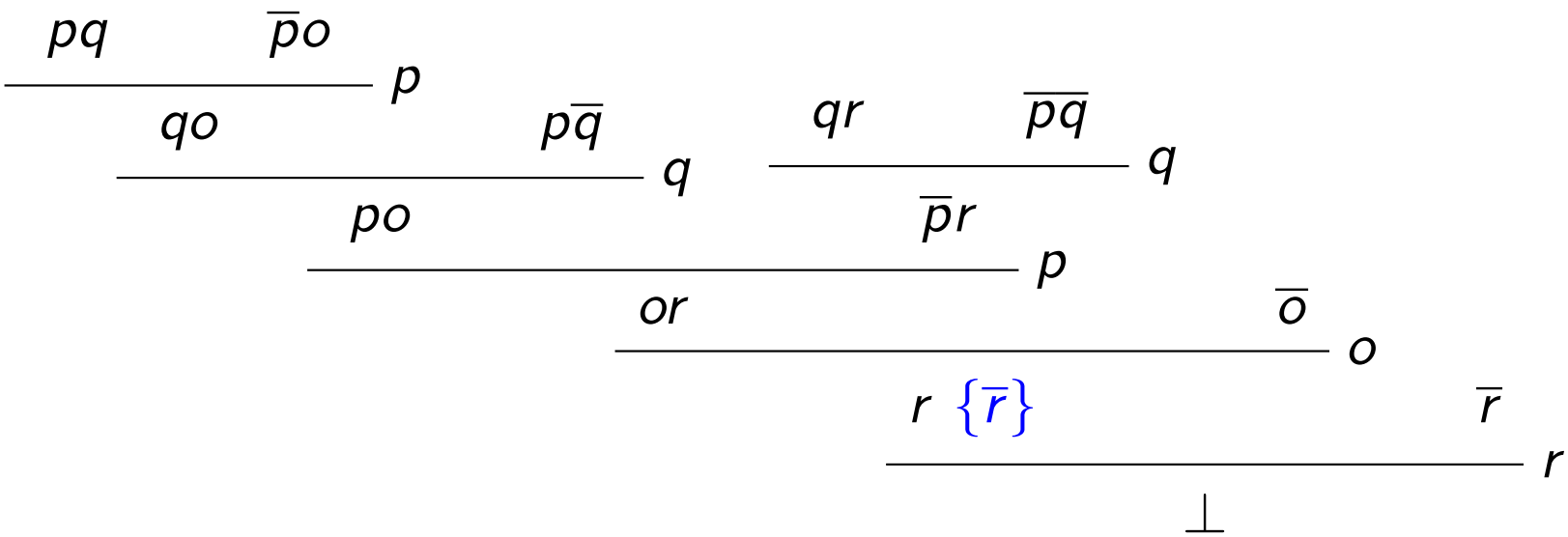
# RecyclePivots

## Example



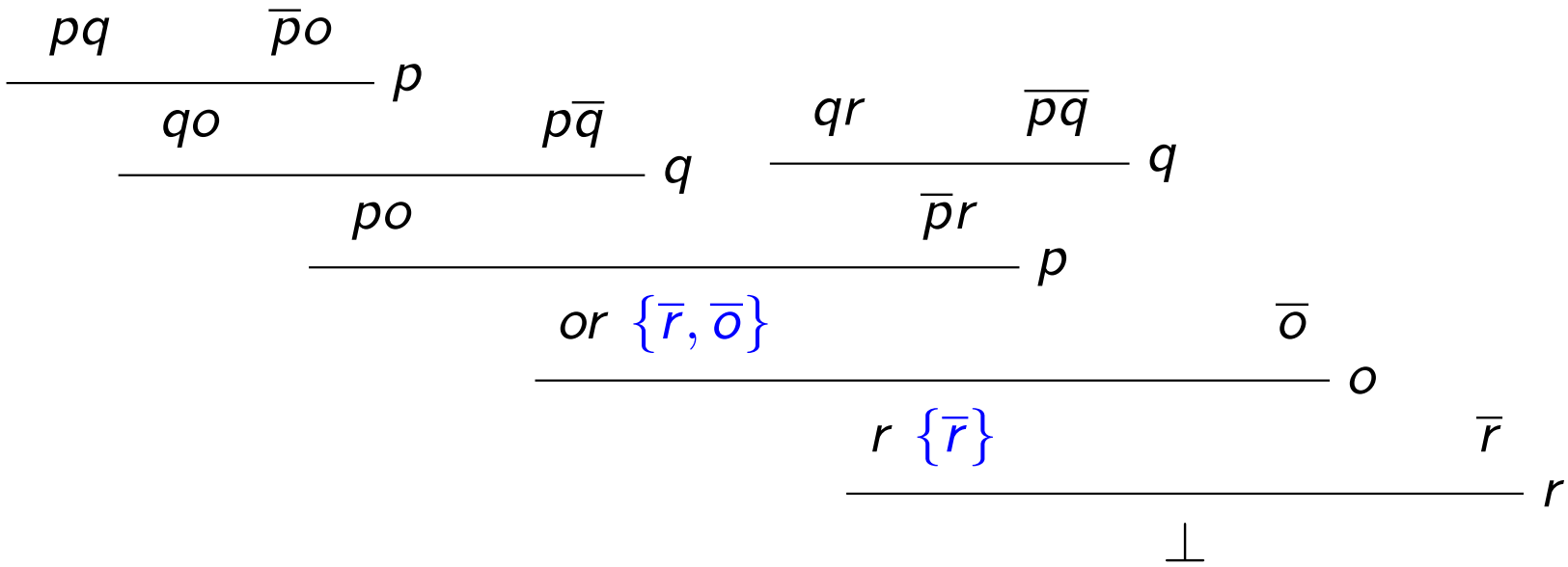
# RecyclePivots

## Example



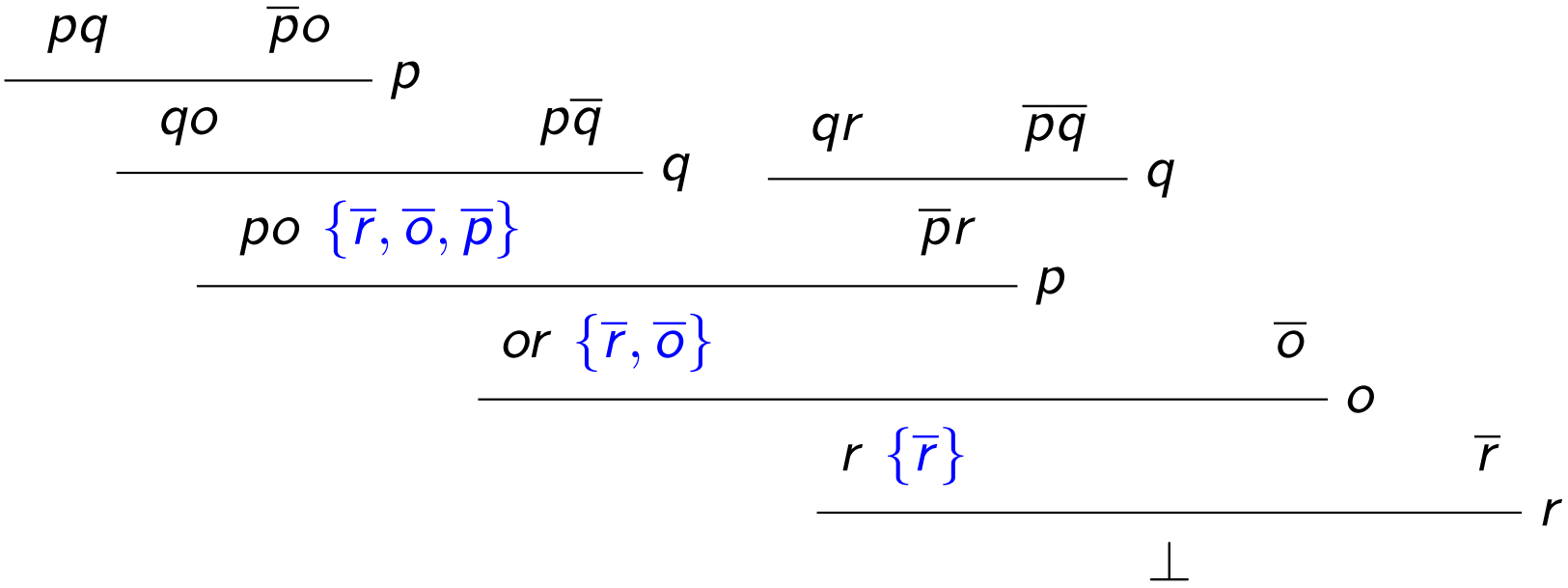
# RecyclePivots

## Example



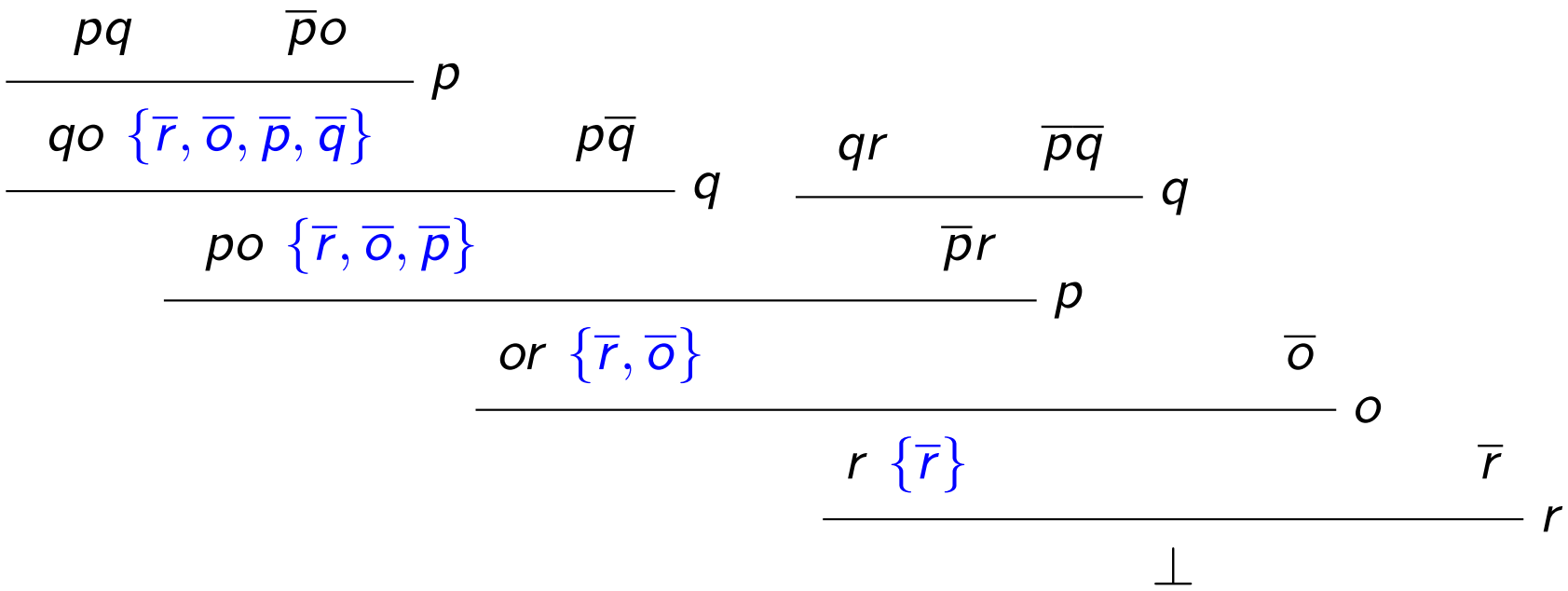
# RecyclePivots

## Example



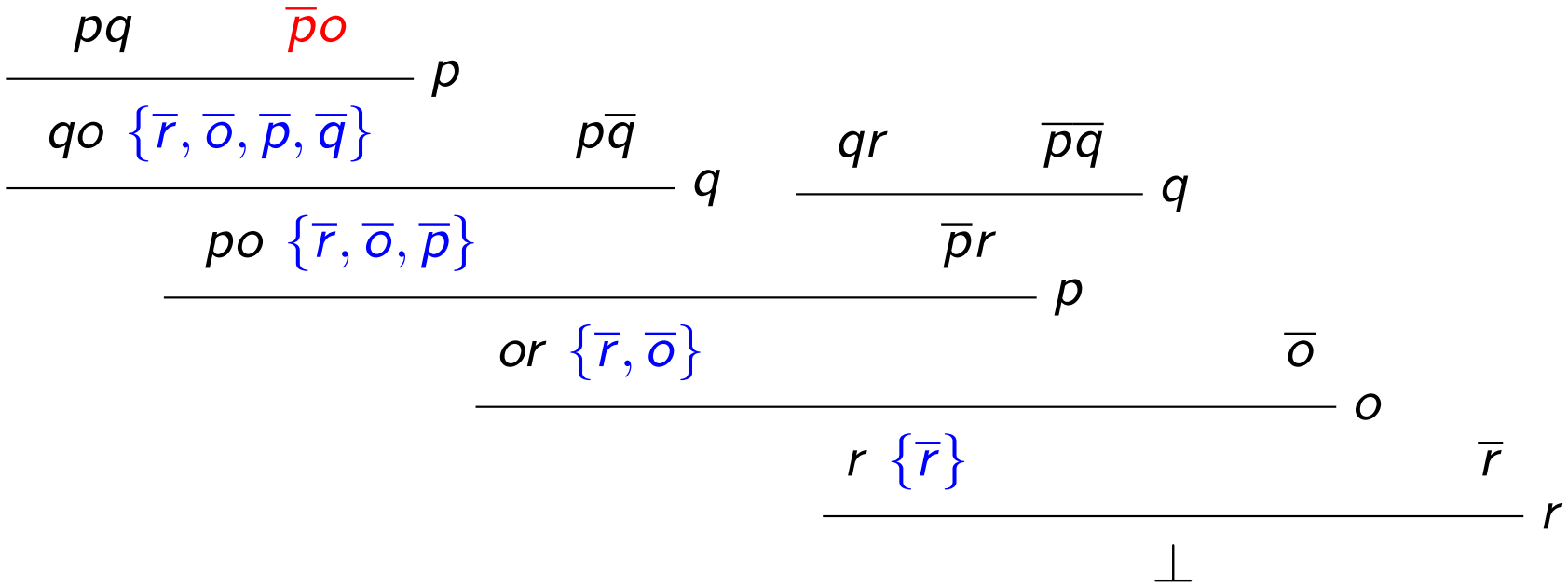
# RecyclePivots

## Example



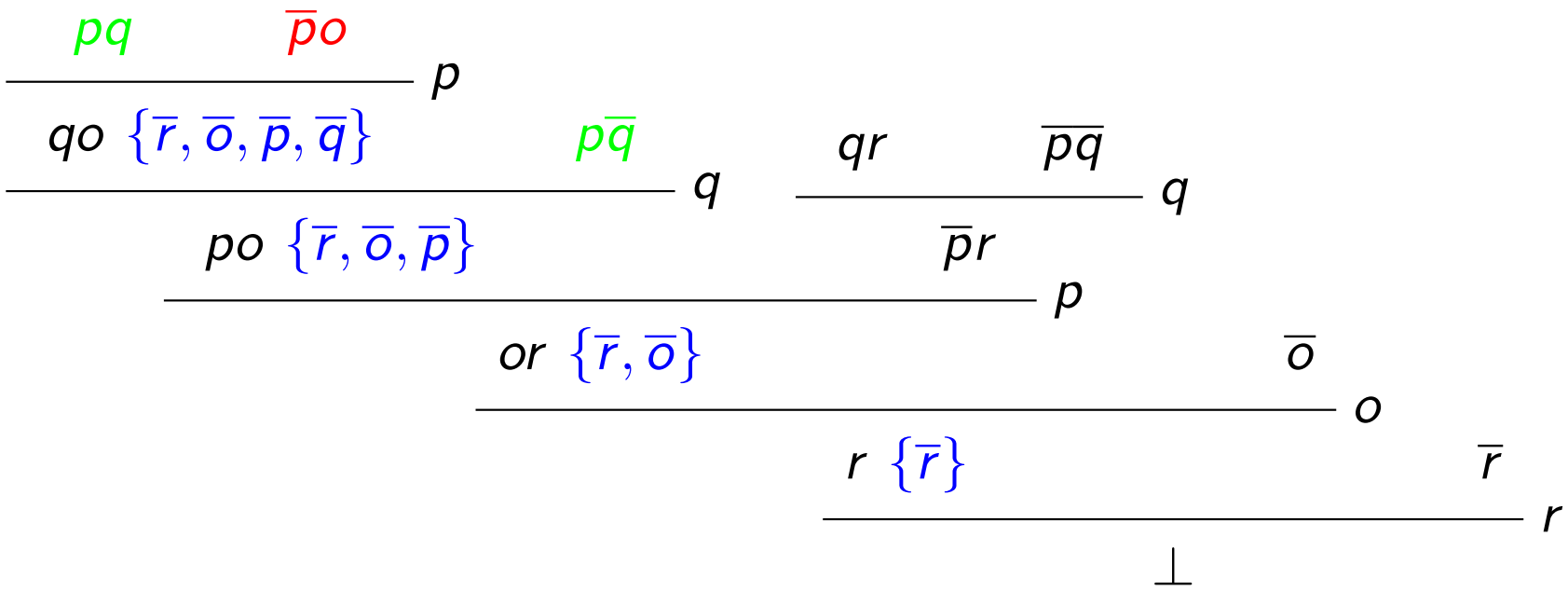
# RecyclePivots

## Example



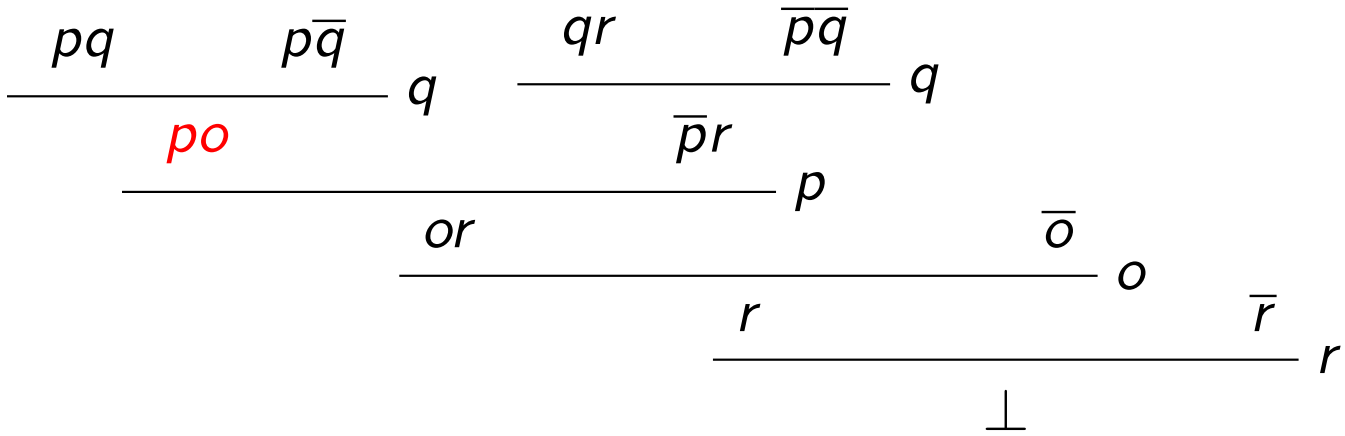
# RecyclePivots

## Example



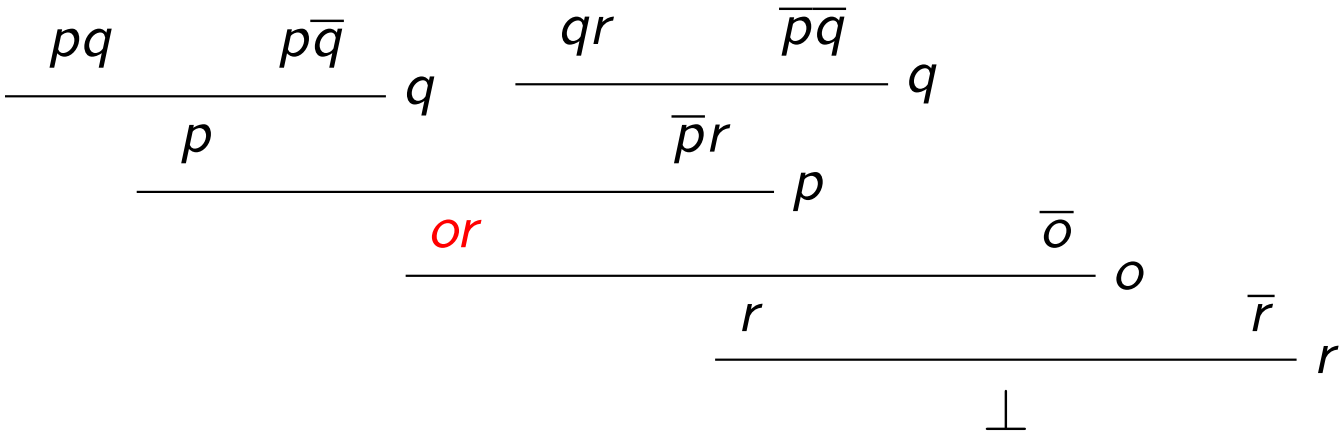
# RecyclePivots

## Example



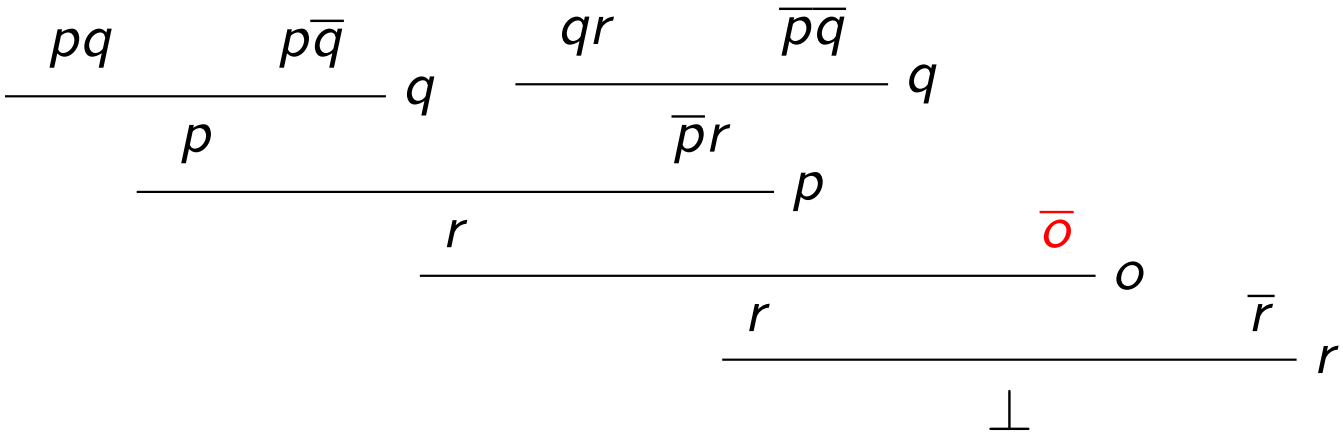
# RecyclePivots

## Example



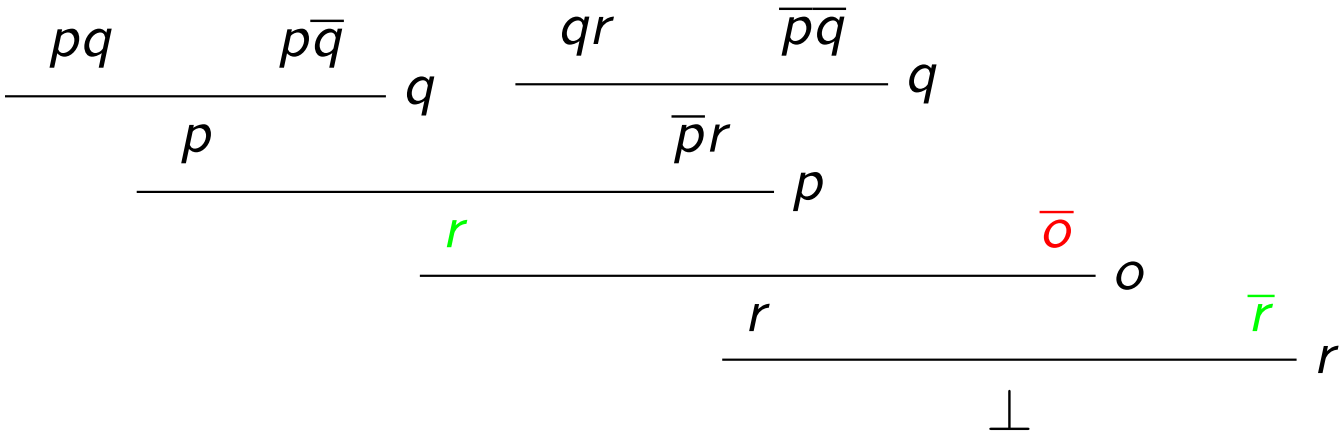
# RecyclePivots

## Example



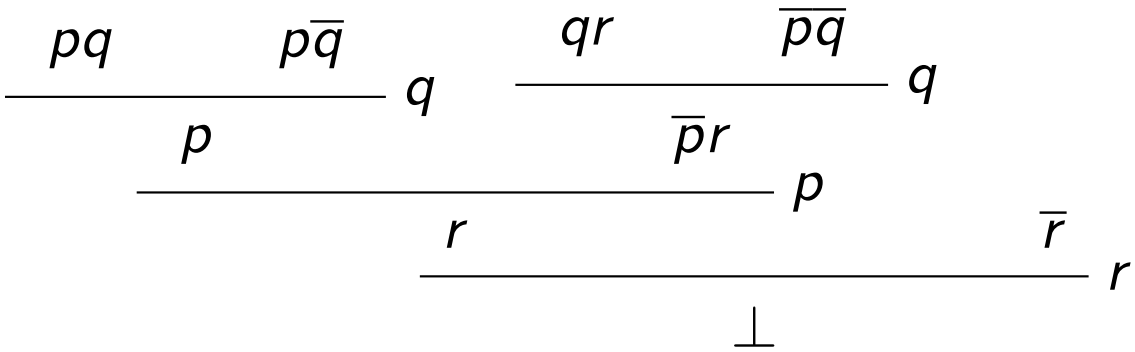
# RecyclePivots

## Example



# RecyclePivots

## Example



# Outline

# Outline

# Transformation Framework

## Features

- Local rewriting rules

# Transformation Framework

## Features

- Local rewriting rules
  - B reduction
  - A perturbation

# Transformation Framework

## Features

- Local rewriting rules
  - B reduction
  - A perturbation

- Rule context

$$\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\,p \qquad \overline{q}E}{CDE}\,q$$

# Transformation Framework

## Features

- Local rewriting rules
  - B reduction
  - A perturbation
- Rule context

$$\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\,p \qquad \overline{q}E}{CDE}\,q$$

- Exhaustiveness up to symmetry

- B rules

| | |
|---|---|
| $B1$ | $\dfrac{\dfrac{pqC \qquad \overline{p}qD}{qCD}\, p \qquad p\overline{q}E}{pCDE}\, q \;\Rightarrow\; \dfrac{pqC \qquad p\overline{q}E}{pCE}\, q$ |

- B rules

| | |
|---|---|
| $B1$ | $\dfrac{\dfrac{pqC \qquad \overline{p}qD}{qCD}\, p \qquad p\overline{q}E}{pCDE}\, q \;\Rightarrow\; \dfrac{pqC \qquad p\overline{q}E}{pCE}\, q$ |

  - Redundancy as reintroduction variable after elimination

# Transformation Framework

## Local rewriting rules

- B rules

| | |
|---|---|
| $B1$ | $\dfrac{\dfrac{pqC \qquad \overline{p}qD}{qCD}\, p \qquad p\overline{q}E}{pCDE}\, q \;\Rightarrow\; \dfrac{pqC \qquad p\overline{q}E}{pCE}\, q$ |

- Redundancy as reintroduction variable after elimination
- Subproof simplification

- B rules

| | |
|---|---|
| $B1$ | $\dfrac{\dfrac{pqC \qquad \overline{p}qD}{qCD}\, p \qquad p\overline{q}E}{pCDE}\, q \;\Rightarrow\; \dfrac{pqC \qquad p\overline{q}E}{pCE}\, q$ |

- Redundancy as reintroduction variable after elimination
- Subproof simplification
- Subproof root strengthening

- A rules

| | |
|---|---|
| $A2$ | $\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\,p \qquad \overline{q}E}{CDE}\,q \;\Rightarrow\; \dfrac{\dfrac{pqC \qquad \overline{q}E}{pCE}\,q \qquad \overline{p}D}{CDE}\,p$ |

- A rules

| | |
|---|---|
| $A2$ | $\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\,p \qquad \overline{q}E}{CDE}\,q \;\Rightarrow\; \dfrac{\dfrac{pqC \qquad \overline{q}E}{pCE}\,q \qquad \overline{p}D}{CDE}\,p$ |

  - Pivots swapping

- A rules

| | |
|---|---|
| $A2$ | $\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\, p \qquad \overline{q}E}{CDE}\, q \;\Rightarrow\; \dfrac{\dfrac{pqC \qquad \overline{q}E}{pCE}\, q \qquad \overline{p}D}{CDE}\, p$ |

  - Pivots swapping
  - Topology perturbation

- A rules

| | |
|---|---|
| $A2$ | $\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\, p \qquad \overline{q}E}{CDE}\, q \quad \Rightarrow \quad \dfrac{\dfrac{pqC \qquad \overline{q}E}{pCE}\, q \qquad \overline{p}D}{CDE}\, p$ |

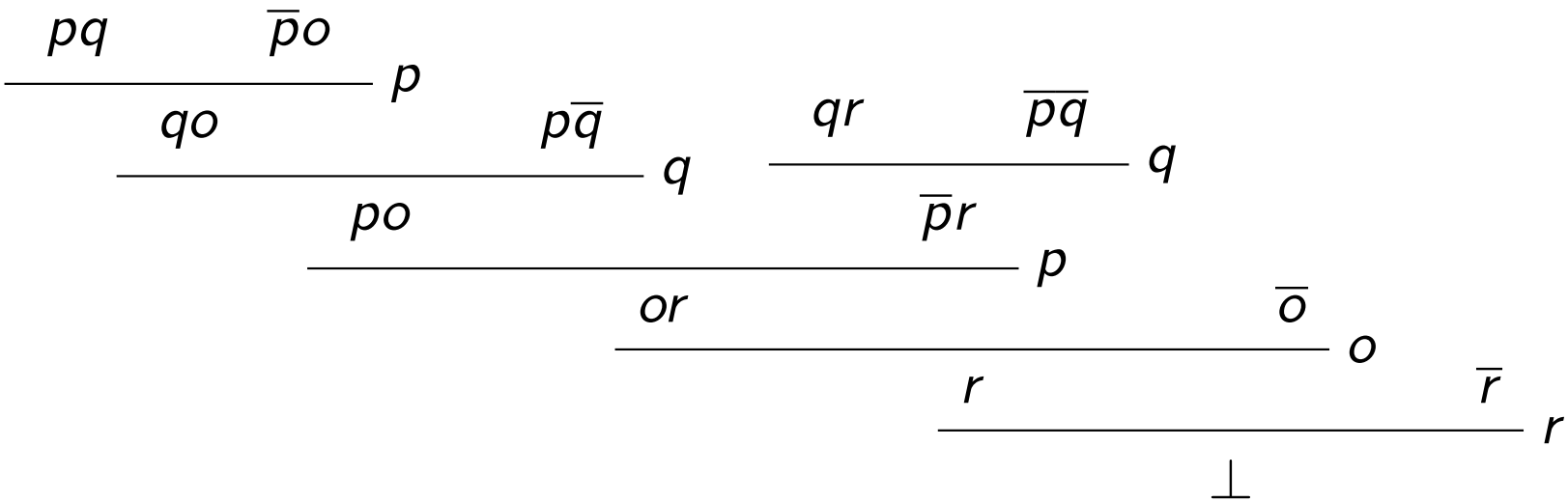
  - Pivots swapping
  - Topology perturbation
  - Redundancies exposure

# Local rewriting rules

| | |
|---|---|
| $A1$ | $\dfrac{\dfrac{pqC \quad \overline{p}qD}{qCD}\,p \quad \overline{q}E}{CDE}\,q \Rightarrow \dfrac{pqC \quad \overline{q}E}{pCE}\,q \quad \dfrac{\overline{q}E \quad \overline{p}qD}{\overline{p}DE}\,q \quad p$ |
| $A2$ | $\dfrac{\dfrac{pqC \quad \overline{p}D}{qCD}\,p \quad \overline{q}E}{CDE}\,q \Rightarrow \dfrac{pqC \quad \overline{q}E}{pCE}\,q \quad \overline{p}D \quad p$ |
| $B1$ | $\dfrac{\dfrac{pqC \quad \overline{p}qD}{qCD}\,p \quad p\overline{q}E}{pCDE}\,q \Rightarrow \dfrac{pqC \quad p\overline{q}E}{pCE}\,q$ |
| $B2$ | $\dfrac{\dfrac{pqC \quad \overline{p}D}{qDC}\,p \quad p\overline{q}E}{pCDE}\,q \Rightarrow \dfrac{pqC \quad p\overline{q}E}{pCE}\,q \quad \overline{p}D \quad p$ |
| $B2'$ | $\dfrac{\dfrac{pqC \quad \overline{p}D}{qDC}\,p \quad p\overline{q}E}{pCDE}\,q \Rightarrow \dfrac{pqC \quad p\overline{q}E}{pCE}\,q$ |
| $B3$ | $\dfrac{\dfrac{pqC \quad \overline{p}D}{qCD}\,p \quad \overline{p}\overline{q}E}{\overline{p}CDE}\,q \Rightarrow \overline{p}D$ |

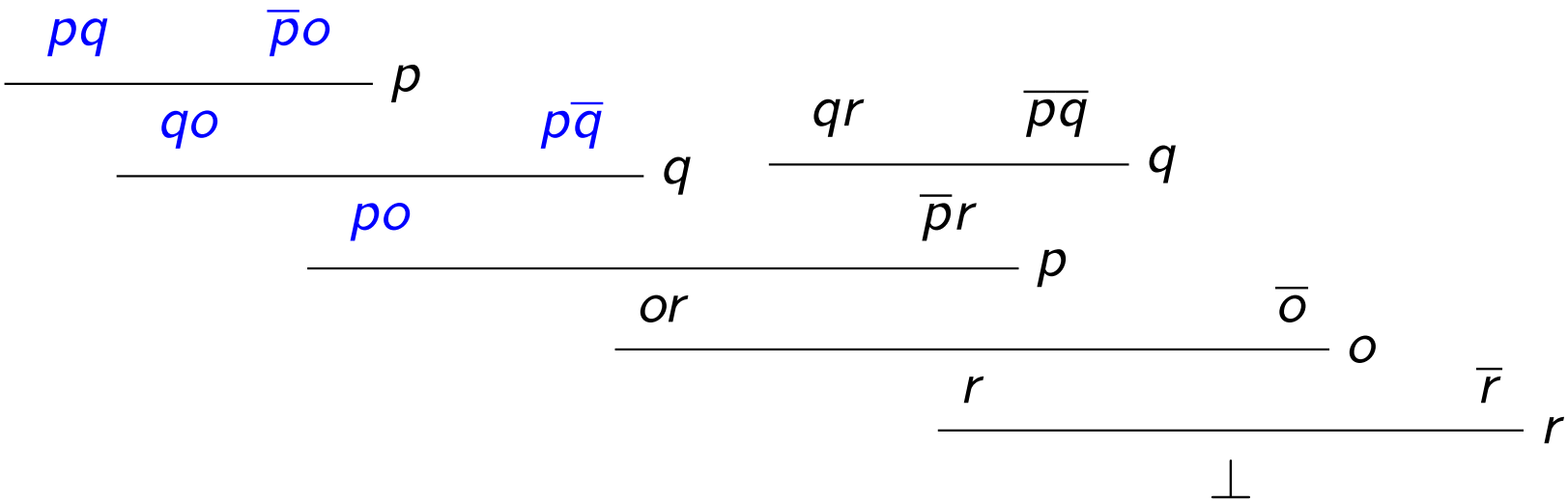
# Rule-based Approach

## Example



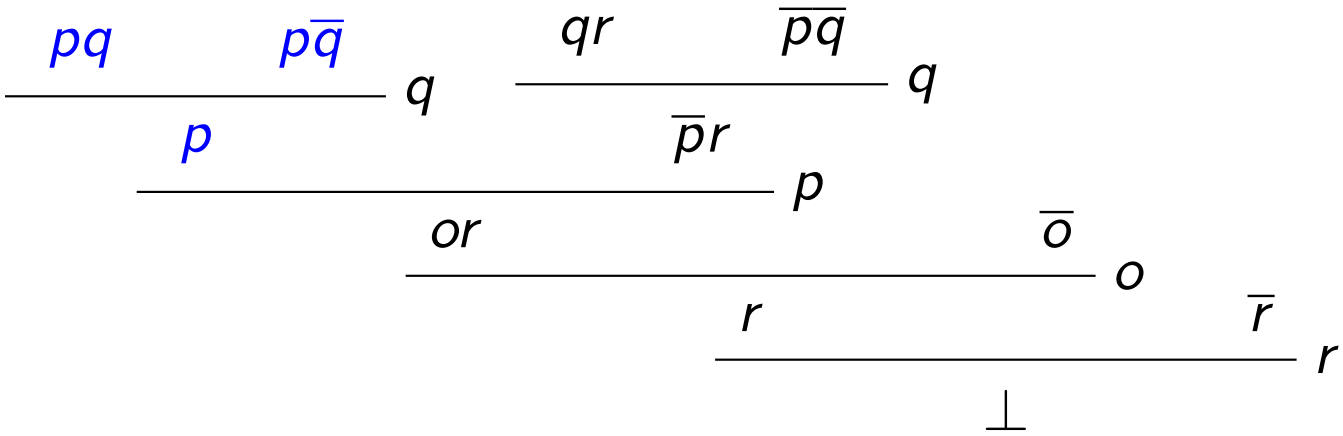
# Rule-based Approach

## Example



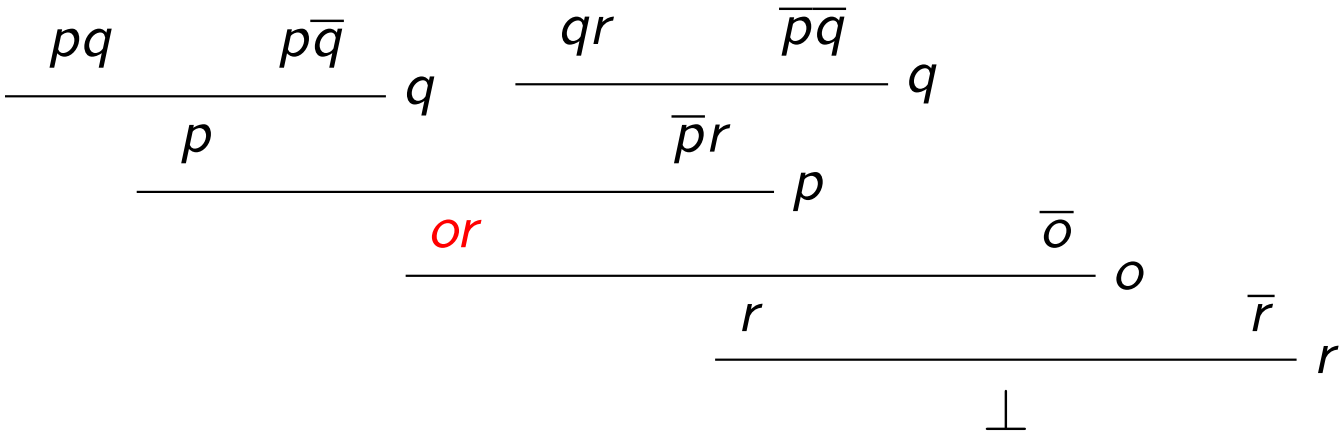
# Rule-based Approach

## Example



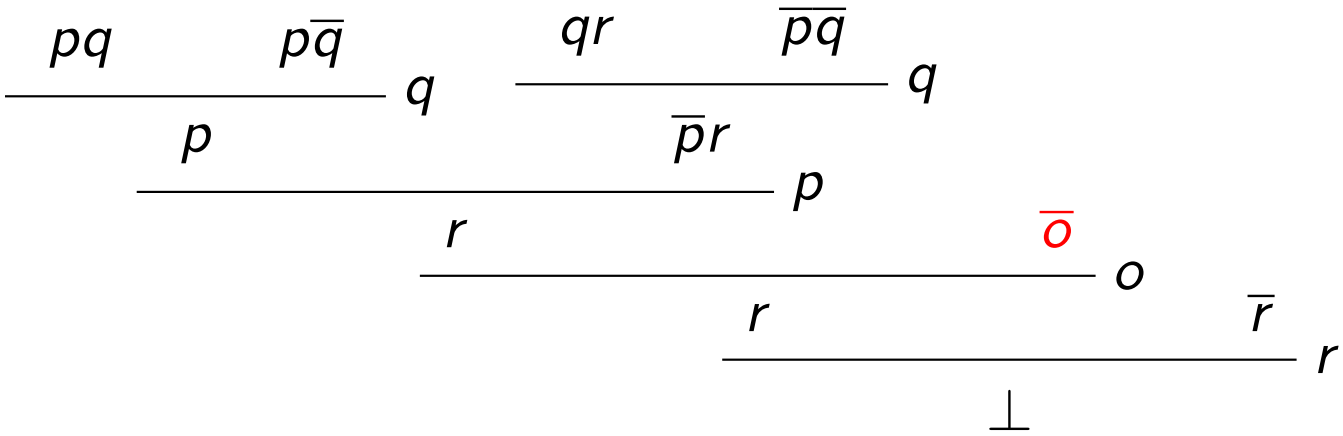
# Rule-based Approach

## Example



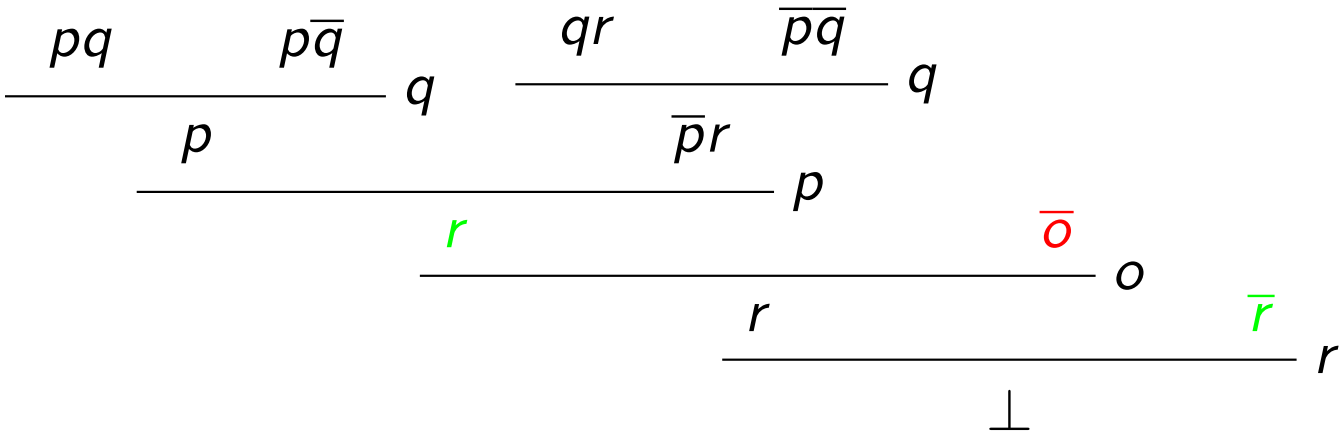
# Rule-based Approach

## Example



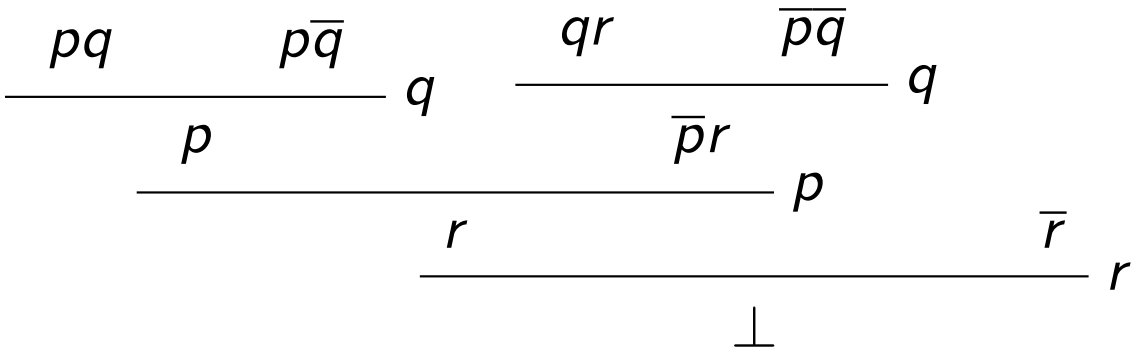
# Rule-based Approach

## Example



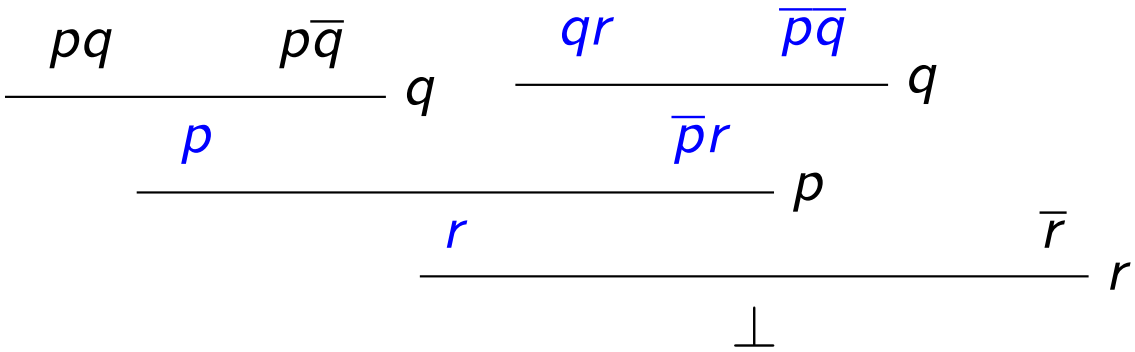
# Rule-based Approach

## Example



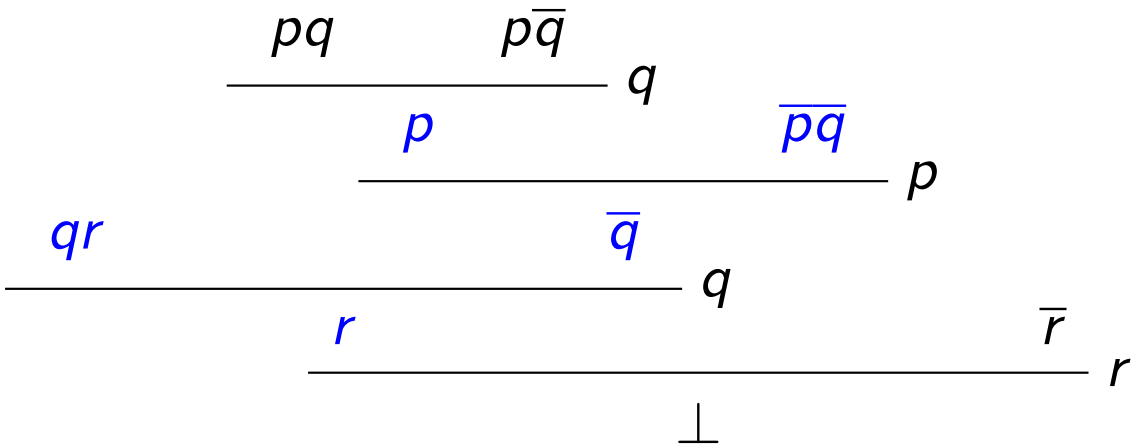
# Rule-based Approach

## Example



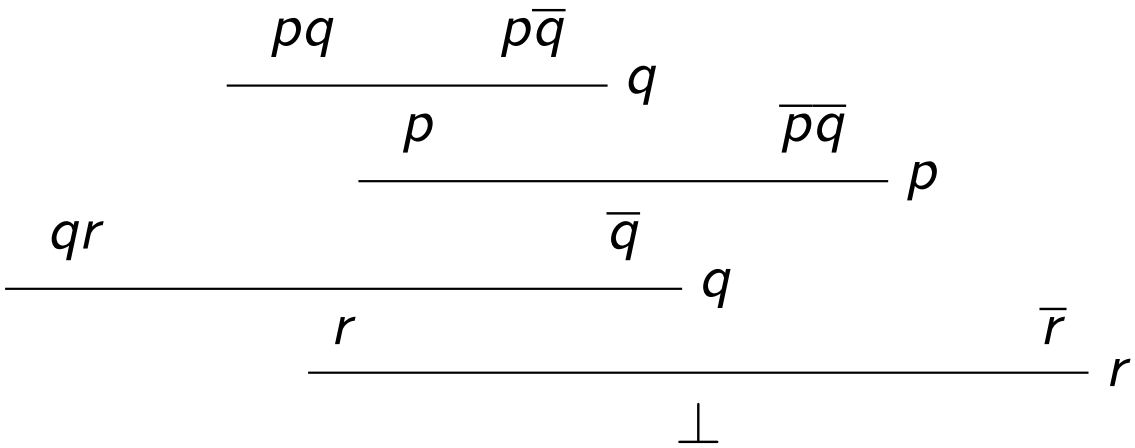
# Rule-based Approach

## Example



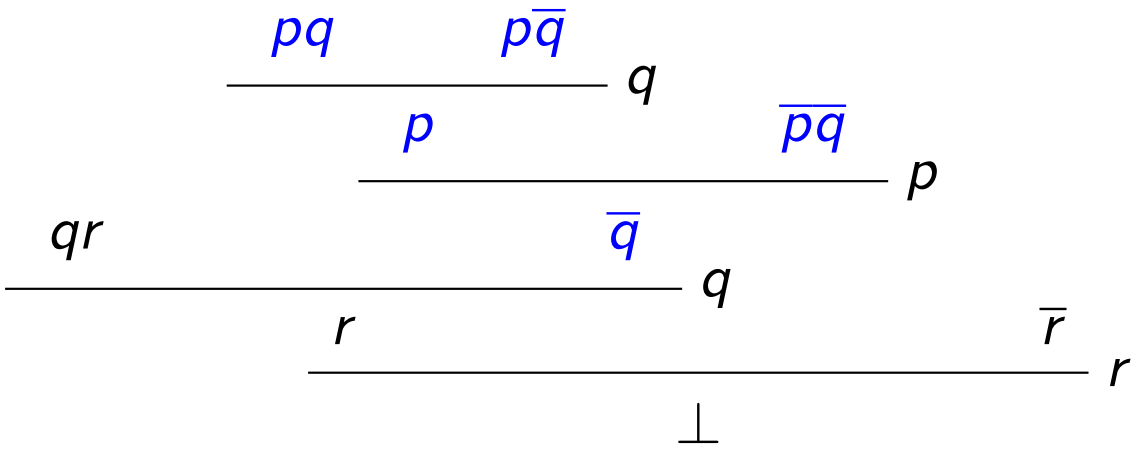
# Rule-based Approach

## Example



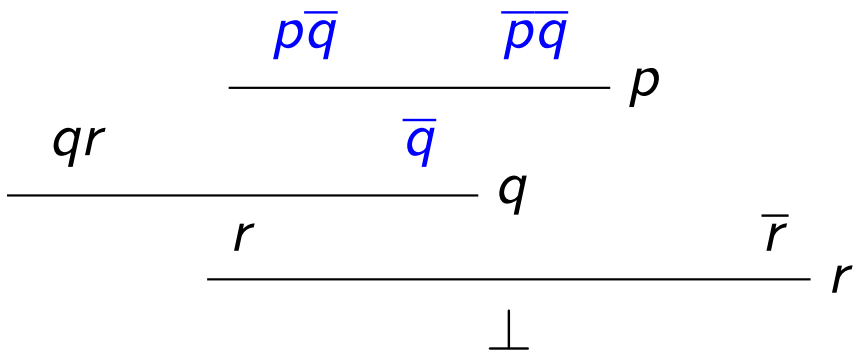
# Rule-based Approach

## Example



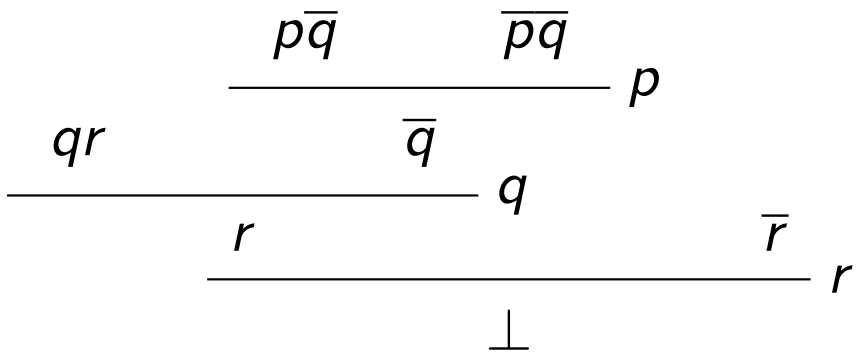
# Rule-based Approach

## Example



# Rule-based Approach

## Example



# Comparison

- RecyclePivots

# Comparison

- RecyclePivots
  - **Pros**
    Global information
    Fast and effective
  - **Cons**
    Cannot expose redundancies

# Comparison

- RecyclePivots
  - **Pros**
    Global information
    Fast and effective
  - **Cons**
    Cannot expose redundancies
- Rule-based approach

# Comparison

- RecyclePivots
  - **Pros**
    Global information
    Fast and effective
  - **Cons**
    Cannot expose redundancies
- Rule-based approach
  - **Pros**
    Flexibility in rules application
    Flexibility in amount of transformation
    Can expose redundancies
  - **Cons**
    Local information

# Outline

# Implementation

## A Simple Algorithm

- Based on a sequence of proof traversals (e.g. topological order)

# Implementation

## A Simple Algorithm

- Based on a sequence of proof traversals (e.g. topological order)
- Parameterized in number of traversals and time limit

# Implementation

## A Simple Algorithm

- Based on a sequence of proof traversals (e.g. topological order)
- Parameterized in number of traversals and time limit
- Examination non-leaf clauses

# Implementation

## A Simple Algorithm

- Based on a sequence of proof traversals (e.g. topological order)
- Parameterized in number of traversals and time limit
- Examination non-leaf clauses
  - Pivot in both antecedents $\rightarrow$ update, match context, apply rule

$$\frac{\dfrac{qC'D' \qquad \overline{q}E'}{CDE}\,q}{} \quad \Rightarrow \quad \frac{\dfrac{qC'D' \qquad \overline{q}E'}{C'D'E'}\,q}{} \quad \Rightarrow \quad \frac{\dfrac{\dfrac{pqC' \qquad \overline{p}D'}{qC'D'}\,p \qquad \overline{q}E'}{C'D'E'}\,q}{}$$

# Implementation

## A Simple Algorithm

- Based on a sequence of proof traversals (e.g. topological order)
- Parameterized in number of traversals and time limit
- Examination non-leaf clauses
  - Pivot in both antecedents $\rightarrow$ update, match context, apply rule

$$\frac{\dfrac{qC'D' \qquad \overline{q}E'}{CDE}\,q}{} \quad \Rightarrow \quad \frac{\dfrac{qC'D' \qquad \overline{q}E'}{C'D'E'}\,q}{} \quad \Rightarrow \quad \frac{\dfrac{\dfrac{pqC' \qquad \overline{p}D'}{qC'D'}\,p \qquad \overline{q}E'}{C'D'E'}\,q}{}$$

  - Pivot not in both antecedents $\rightarrow$ remove resolution step

$$\frac{\dfrac{C'D' \qquad \overline{q}E'}{CDE}\,q}{} \quad \Rightarrow \quad C'D'$$

# Implementation

## A Simple Algorithm

- Based on a sequence of proof traversals (e.g. topological order)
- Parameterized in number of traversals and time limit
- Examination non-leaf clauses
  - Pivot in both antecedents $\rightarrow$ update, match context, apply rule

$$\frac{\dfrac{qC'D' \qquad \overline{q}E'}{CDE}\,q}{} \quad \Rightarrow \quad \frac{\dfrac{qC'D' \qquad \overline{q}E'}{C'D'E'}\,q}{} \quad \Rightarrow \quad \frac{\dfrac{\dfrac{pqC' \qquad \overline{p}D'}{qC'D'}\,p \qquad \overline{q}E'}{C'D'E'}\,q}{}$$

  - Pivot not in both antecedents $\rightarrow$ remove resolution step

$$\frac{\dfrac{C'D' \qquad \overline{q}E'}{CDE}\,q}{} \quad \Rightarrow \quad C'D'$$

- Easy combination with RecyclePivots

# Evaluation

## Framework and Benchmarks

- opensmt

# Evaluation

## Framework and Benchmarks

- opensmt
  - Open-source SMT solver developed at USI
  - Fastest open-source solver in SMT-comp 2009, 2010 for various logics
  - C++ framework for research and experimentation

# Evaluation

## Framework and Benchmarks

- opensmt
  - Open-source SMT solver developed at USI
  - Fastest open-source solver in SMT-comp 2009, 2010 for various logics
  - C++ framework for research and experimentation
- Benchmarks

# Evaluation

## Framework and Benchmarks

- opensmt
  - Open-source SMT solver developed at USI
  - Fastest open-source solver in SMT-comp 2009, 2010 for various logics
  - C++ framework for research and experimentation

- Benchmarks
  - SMT: SMT-LIB library
  - SAT: SAT competition
  - Academic and industrial problems

# Combined Approach Evaluation

## Experimental results over SMT: QF_UF, QF_IDL, QF_LRA, QF_RDL

| | # | $Avg_{nodes}$ | $Avg_{edges}$ | $Avg_{core}$ | $T(s)$ | $Max_{nodes}$ | $Max_{edges}$ | $Max_{core}$ |
|---|---|---|---|---|---|---|---|---|
| RP | 1370 | 6.7% | 7.5% | 1.3% | 1.7 | 65.1% | 68.9% | 39.1% |
| Ratio | | | | | | | | |
| 0.01 | 1366 | 8.9% | 10.7% | 1.4% | 3.4 | 66.3% | 70.2% | 45.7% |
| 0.025 | 1366 | 9.8% | 11.9% | 1.5% | 3.6 | 77.2% | 79.9% | 45.7% |
| 0.05 | 1366 | 10.7% | 13.0% | 1.6% | 4.1 | 78.5% | 81.2% | 45.7% |
| 0.075 | 1366 | 11.4% | 13.8% | 1.7% | 4.5 | 78.5% | 81.2% | 45.7% |
| 0.1 | 1364 | 11.8% | 14.4% | 1.7% | 5.0 | 78.8% | 83.6% | 45.7% |
| 0.25 | 1359 | 13.6% | 16.6% | 1.9% | 7.6 | 79.6% | 84.4% | 45.7% |
| 0.5 | 1348 | 15.0% | 18.4% | 2.0% | 11.5 | 79.1% | 85.2% | 45.7% |
| 0.75 | 1341 | 16.0% | 19.5% | 2.1% | 15.1 | 79.9% | 86.1% | 45.7% |
| 1 | 1337 | 16.7% | 20.4% | 2.2% | 18.8 | 79.9% | 86.1% | 45.7% |

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- $Avg_{nodes}$, $Avg_{edges}$, $Avg_{core}$ — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- $Max_{nodes}$, $Max_{edges}$, $Max_{core}$ — max reduction in proof size

# Combined Approach Evaluation

## Experimental results over SMT: QF_UF, QF_IDL, QF_LRA, QF_RDL

| | # | $Avg_{nodes}$ | $Avg_{edges}$ | $Avg_{core}$ | $T(s)$ | $Max_{nodes}$ | $Max_{edges}$ | $Max_{core}$ |
|---|---|---|---|---|---|---|---|---|
| RP | 1370 | 6.7% | 7.5% | 1.3% | 1.7 | 65.1% | 68.9% | 39.1% |
| Ratio | | | | | | | | |
| 0.01 | 1366 | 8.9% | 10.7% | 1.4% | 3.4 | 66.3% | 70.2% | 45.7% |
| 0.025 | 1366 | 9.8% | 11.9% | 1.5% | 3.6 | 77.2% | 79.9% | 45.7% |
| 0.05 | 1366 | 10.7% | 13.0% | 1.6% | 4.1 | 78.5% | 81.2% | 45.7% |
| 0.075 | 1366 | 11.4% | 13.8% | 1.7% | 4.5 | 78.5% | 81.2% | 45.7% |
| 0.1 | 1364 | 11.8% | 14.4% | 1.7% | 5.0 | 78.8% | 83.6% | 45.7% |
| 0.25 | 1359 | 13.6% | 16.6% | 1.9% | 7.6 | 79.6% | 84.4% | 45.7% |
| 0.5 | 1348 | 15.0% | 18.4% | 2.0% | 11.5 | 79.1% | 85.2% | 45.7% |
| 0.75 | 1341 | 16.0% | 19.5% | 2.1% | 15.1 | 79.9% | 86.1% | 45.7% |
| 1 | 1337 | 16.7% | 20.4% | 2.2% | 18.8 | 79.9% | 86.1% | 45.7% |

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- $Avg_{nodes}$, $Avg_{edges}$, $Avg_{core}$ — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- $Max_{nodes}$, $Max_{edges}$, $Max_{core}$ — max reduction in proof size

# Combined Approach Evaluation

## Experimental results over SMT: QF_UF, QF_IDL, QF_LRA, QF_RDL

| | # | $Avg_{nodes}$ | $Avg_{edges}$ | $Avg_{core}$ | $T(s)$ | $Max_{nodes}$ | $Max_{edges}$ | $Max_{core}$ |
|---|---|---|---|---|---|---|---|---|
| RP | 1370 | 6.7% | 7.5% | 1.3% | 1.7 | 65.1% | 68.9% | 39.1% |
| Ratio | | | | | | | | |
| 0.01 | 1366 | 8.9% | 10.7% | 1.4% | 3.4 | 66.3% | 70.2% | 45.7% |
| 0.025 | 1366 | 9.8% | 11.9% | 1.5% | 3.6 | 77.2% | 79.9% | 45.7% |
| 0.05 | 1366 | 10.7% | 13.0% | 1.6% | 4.1 | 78.5% | 81.2% | 45.7% |
| 0.075 | 1366 | 11.4% | 13.8% | 1.7% | 4.5 | 78.5% | 81.2% | 45.7% |
| 0.1 | 1364 | 11.8% | 14.4% | 1.7% | 5.0 | 78.8% | 83.6% | 45.7% |
| 0.25 | 1359 | 13.6% | 16.6% | 1.9% | 7.6 | 79.6% | 84.4% | 45.7% |
| 0.5 | 1348 | 15.0% | 18.4% | 2.0% | 11.5 | 79.1% | 85.2% | 45.7% |
| 0.75 | 1341 | 16.0% | 19.5% | 2.1% | 15.1 | 79.9% | 86.1% | 45.7% |
| 1 | 1337 | 16.7% | 20.4% | 2.2% | 18.8 | 79.9% | 86.1% | 45.7% |

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- $Avg_{nodes}$, $Avg_{edges}$, $Avg_{core}$ — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- $Max_{nodes}$, $Max_{edges}$, $Max_{core}$ — max reduction in proof size

# Combined Approach Evaluation

## Experimental results over SAT

| | # | $Avg_{nodes}$ | $Avg_{edges}$ | $Avg_{core}$ | $T(s)$ | $Max_{nodes}$ | $Max_{edges}$ | $Max_{core}$ |
|---|---|---|---|---|---|---|---|---|
| RP | 25 | 5.9% | 6.5% | 1.7% | 10.8 | 33.1% | 33.4% | 30.3% |
| *Ratio* | | | | | | | | |
| 0.01 | 25 | 6.8% | 7.9% | 1.7% | 32.3 | 34.0% | 34.4% | 30.5% |
| 0.025 | 25 | 6.8% | 7.9% | 1.7% | 32.3 | 34.0% | 34.4% | 30.5% |
| 0.05 | 25 | 7.0% | 8.2% | 1.8% | 40.0 | 34.0% | 34.4% | 30.5% |
| 0.075 | 25 | 7.2% | 8.4% | 1.8% | 49.3 | 34.7% | 35.1% | 30.5% |
| 0.1 | 25 | 7.3% | 8.4% | 1.8% | 60.2 | 34.7% | 35.1% | 30.5% |
| 0.25 | 25 | 7.6% | 8.8% | 1.9% | 125.3 | 39.8% | 40.6% | 31.7% |
| 0.5 | 25 | 7.8% | 9.1% | 1.9% | 243.5 | 41.0% | 41.9% | 32.1% |
| 0.75 | 25 | 7.9% | 9.3% | 1.9% | 360.0 | 41.6% | 42.6% | 32.1% |
| 1 | 23 | 8.4% | 9.9% | 2.1% | 175.6 | 33.1% | 33.4% | 30.6% |

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- $Avg_{nodes}$, $Avg_{edges}$, $Avg_{core}$ — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- $Max_{nodes}$, $Max_{edges}$, $Max_{core}$ — max reduction in proof size

# Combined Approach Evaluation

## Experimental results over SAT

| | # | $Avg_{nodes}$ | $Avg_{edges}$ | $Avg_{core}$ | $T(s)$ | $Max_{nodes}$ | $Max_{edges}$ | $Max_{core}$ |
|---|---|---|---|---|---|---|---|---|
| RP | 25 | 5.9% | 6.5% | 1.7% | 10.8 | 33.1% | 33.4% | 30.3% |
| *Ratio* | | | | | | | | |
| 0.01 | 25 | 6.8% | 7.9% | 1.7% | 32.3 | 34.0% | 34.4% | 30.5% |
| 0.025 | 25 | 6.8% | 7.9% | 1.7% | 32.3 | 34.0% | 34.4% | 30.5% |
| 0.05 | 25 | 7.0% | 8.2% | 1.8% | 40.0 | 34.0% | 34.4% | 30.5% |
| 0.075 | 25 | 7.2% | 8.4% | 1.8% | 49.3 | 34.7% | 35.1% | 30.5% |
| 0.1 | 25 | 7.3% | 8.4% | 1.8% | 60.2 | 34.7% | 35.1% | 30.5% |
| 0.25 | 25 | 7.6% | 8.8% | 1.9% | 125.3 | 39.8% | 40.6% | 31.7% |
| 0.5 | 25 | 7.8% | 9.1% | 1.9% | 243.5 | 41.0% | 41.9% | 32.1% |
| 0.75 | 25 | 7.9% | 9.3% | 1.9% | 360.0 | 41.6% | 42.6% | 32.1% |
| 1 | 23 | 8.4% | 9.9% | 2.1% | 175.6 | 33.1% | 33.4% | 30.6% |

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- $Avg_{nodes}$, $Avg_{edges}$, $Avg_{core}$ — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- $Max_{nodes}$, $Max_{edges}$, $Max_{core}$ — max reduction in proof size

# Combined Approach Evaluation

## Experimental results over SAT

| | # | $Avg_{nodes}$ | $Avg_{edges}$ | $Avg_{core}$ | $T(s)$ | $Max_{nodes}$ | $Max_{edges}$ | $Max_{core}$ |
|---|---|---|---|---|---|---|---|---|
| RP | 25 | 5.9% | 6.5% | 1.7% | 10.8 | 33.1% | 33.4% | 30.3% |
| *Ratio* | | | | | | | | |
| 0.01 | 25 | 6.8% | 7.9% | 1.7% | 32.3 | 34.0% | 34.4% | 30.5% |
| 0.025 | 25 | 6.8% | 7.9% | 1.7% | 32.3 | 34.0% | 34.4% | 30.5% |
| 0.05 | 25 | 7.0% | 8.2% | 1.8% | 40.0 | 34.0% | 34.4% | 30.5% |
| 0.075 | 25 | 7.2% | 8.4% | 1.8% | 49.3 | 34.7% | 35.1% | 30.5% |
| 0.1 | 25 | 7.3% | 8.4% | 1.8% | 60.2 | 34.7% | 35.1% | 30.5% |
| 0.25 | 25 | 7.6% | 8.8% | 1.9% | 125.3 | 39.8% | 40.6% | 31.7% |
| 0.5 | 25 | 7.8% | 9.1% | 1.9% | 243.5 | 41.0% | 41.9% | 32.1% |
| 0.75 | 25 | 7.9% | 9.3% | 1.9% | 360.0 | 41.6% | 42.6% | 32.1% |
| 1 | 23 | 8.4% | 9.9% | 2.1% | 175.6 | 33.1% | 33.4% | 30.6% |

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- $Avg_{nodes}$, $Avg_{edges}$, $Avg_{core}$ — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- $Max_{nodes}$, $Max_{edges}$, $Max_{core}$ — max reduction in proof size

# Outline

# Summary and Future Work

- Summary

# Summary and Future Work

- Summary
  - Rule-based proof reduction framework

# Summary and Future Work

- Summary
  - Rule-based proof reduction framework
  - Pivots redundancies

# Summary and Future Work

- Summary
  - Rule-based proof reduction framework
  - Pivots redundancies
  - Comparison and evaluation

# Summary and Future Work

- Summary
  - Rule-based proof reduction framework
  - Pivots redundancies
  - Comparison and evaluation
- Future Work

# Summary and Future Work

- Summary
  - Rule-based proof reduction framework
  - Pivots redundancies
  - Comparison and evaluation
- Future Work
  - Exploitation of DPLL proof structure

# Summary and Future Work

- Summary
  - Rule-based proof reduction framework
  - Pivots redundancies
  - Comparison and evaluation
- Future Work
  - Exploitation of DPLL proof structure
  - Evaluation on concrete applications (e.g. interpolation)

# Thank you for your attention!